The modified Bessel equation is

$$\begin{aligned}
\frac{d}{dx}\left(x\frac{dy}{dx}\right)-xy-\frac{m^2}{x}y &= 0\\
x\frac{d^2y}{dx^2}+\frac{dy}{dx}-xy-\frac{m^2}{x}y &= 0
\end{aligned}$$

Compare with the standard form (Lea 3.18)

$$\frac{d^2y}{dx^2}+f(x)\frac{dy}{dx}+g(x)y=0$$

Thus

$$f(x)=\frac{1}{x}$$

and (Lea eqn 3.22)

$$\begin{aligned}
W &= W(x_0)\exp\left[-\int_{x_0}^{x}\frac{1}{u}du\right]\\
&= W(x_0)\exp(-\ln x/x_0)=W(x_0)\frac{x_0}{x} \qquad (1)
\end{aligned}$$

Since we found

$$W(x_0)=-\frac{1}{x_0}\qquad \text{for } x_0\gg 1$$

Then

$$W(x)=-\frac{1}{x}\qquad \text{for all}\ \ x.$$

Note that the sign depends on which function is designated number 1. We chose K to be #1.

$$W=K_m'I_m-I_m'K_m=-\frac{1}{x}$$

So

$$W=I_m'K_m-K_m'I_m=+\frac{1}{x}$$

Since the regular Bessel equation just has a change of sign in the last term ($+m^2$ replaces $-m^2$), the Wronskian has the same form. For J and N, we have, for large x

$$
\begin{aligned}
J_m' N_m - J_m N_m' &= \frac{d}{dx}\left(\sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{m\pi}{2} - \frac{\pi}{4}\right)\right)\sqrt{\frac{2}{\pi x}}\sin\left(x - \frac{m\pi}{2} - \frac{\pi}{4}\right) \\
&\quad - \left(\sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{m\pi}{2} - \frac{\pi}{4}\right)\right)\frac{d}{dx}\sqrt{\frac{2}{\pi x}}\sin\left(x - \frac{m\pi}{2} - \frac{\pi}{4}\right) \\
&= \frac{2}{\pi}\left(-\frac{1}{2x^{3/2}}\cos\left(x - \chi_m\right) - \frac{1}{\sqrt{x}}\sin\left(x - \chi_m\right)\right)\frac{1}{\sqrt{x}}\sin\left(x - \chi_m\right) \\
&\quad - \frac{2}{\pi}\frac{1}{\sqrt{x}}\cos\left(x - \chi_m\right)\left(-\frac{1}{2x^{3/2}}\sin\left(x - \chi_m\right) + \frac{1}{\sqrt{x}}\cos\left(x - \chi_m\right)\right) \\
&= \frac{2}{\pi}\left\{-\frac{1}{x}\sin^2\left(x - \chi_m\right) - \frac{1}{x}\cos^2\left(x - \chi_m\right)\right\} = -\frac{2}{\pi x}
\end{aligned}
$$

and so by eqn (1), this is the result for all x. Here again you must take care wth the sign.