Review of E&M as applied to circuits.

Maxwell's equations relate the electric field to its sources:

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\varepsilon_0} \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \end{aligned}$$

By the Helmholtz theorem

$$\vec{E} = \vec{E}_{\text{coul}} + \vec{E}_{\text{ind}}$$

where

$$\vec{\nabla} \times \vec{E}_{\text{coul}} = 0 \quad \text{and} \quad \vec{\nabla} \cdot \vec{E}_{\text{ind}} = 0$$

Thus

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{E}_{\text{coul}} = \frac{\rho}{\varepsilon_0}$$

(charge is the source of Coulomb field) and

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times \vec{E}_{\text{ind}} = -\frac{\partial \vec{B}}{\partial t}$$

(changing $\vec{B}$ is the source of induced field.)

Kirchoff's rules:

For a simple, static circuit, the loop rule is a statement of energy conservation and the junction rule is a statement of charge conservation. However, for time-dependent circuits we have to be more careful. The loop rule says:

The change in potential around any closed curve is zero.

This is true because the electric scalar potential is a scalar field with a single, well-defined value at every point in space. The scalar potential is directly related to the Coulomb field, since

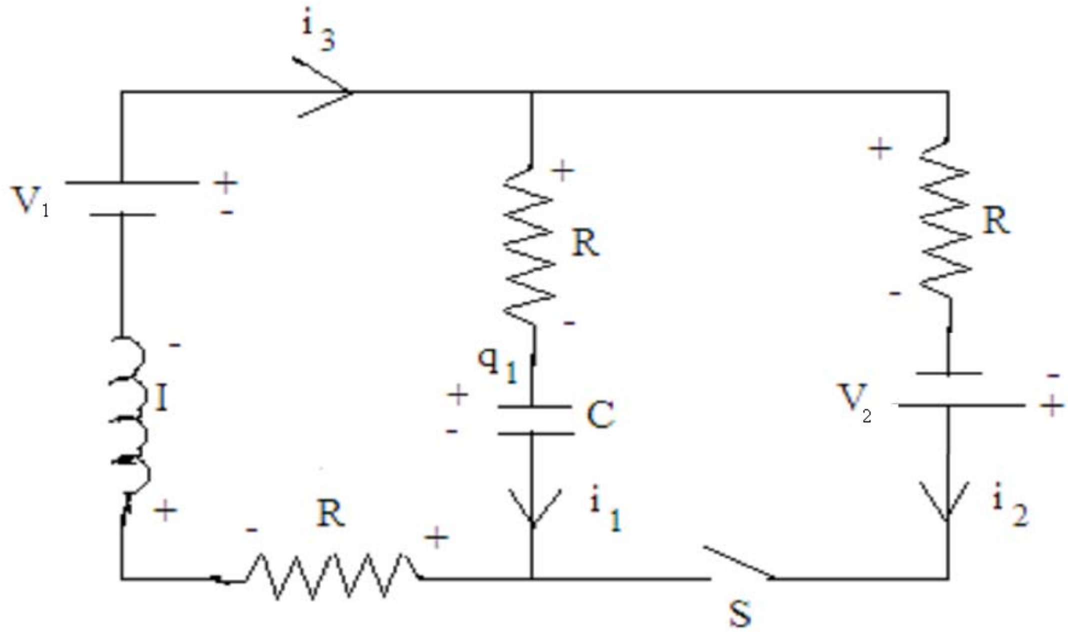
$$\vec{\nabla} \times \vec{E}_{\text{coul}} = 0 \Rightarrow \vec{E}_{\text{coul}} = -\vec{\nabla}\Phi$$

Thus the scalar potential is related to the charge density, because $\vec{E}_{\text{coul}}$ is. The induced field is not related to Φ but to the vector potential $\vec{A}$

$$\begin{aligned} \vec{\nabla} \times \vec{E}_{\text{ind}} &= -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t} \vec{\nabla} \times \vec{A} \\ \vec{E}_{\text{ind}} &= -\frac{\partial}{\partial t} \vec{A} \end{aligned}$$

For a time-dependent circuit, we begin by **defining the charge and current variables**. The current is the flux of current density. Flux of any vector field is a scalar, but it has a sign that is related to direction. If the direction of I is opposite our defined direction, we will simply get a negative value for I. There is no need to attempt to guess the correct direction ahead of time, and for an oscillating current it is pointless.

If the circuit contains capacitors, the current variable(s) will be related to the charge variables through charge conservation.



Junction rule:

Current flowing into any junction equals current flowing out.

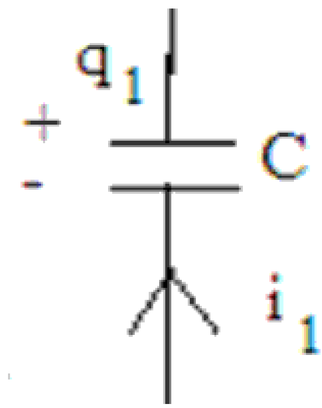
$$I_1 + I_2 = I_3$$

Relation of I to q.

In the diagram above, positive I_1 increases q_1, so

$$I_1 = +\frac{dq_1}{dt}$$

But in the diagram below, positive I_1 reduces q_1, so



$$I_1 = -\frac{dq_1}{dt}$$

The signs are important!

Now what about the signs of the potential differences. The positively charged plate of the capacitor is at the higher potential, and

$$\Delta V_C = \frac{Q}{C}$$

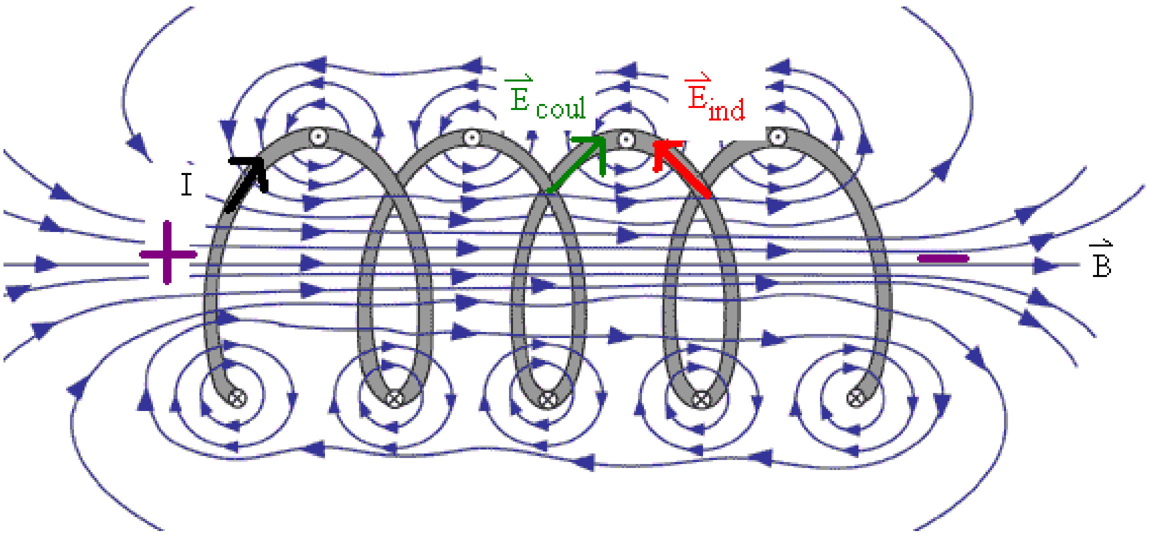
Similarly, the electric field must point in the direction of I to push the current through a resistor. So the higher potential is at the end that current enters the resistor, with

$$\Delta V_R = IR$$

The inductor is more complicated. We model the inductor as a perfectly conducting coil. In the perfect conductor, the total electric field is zero:

$$\vec{E}_{\mathrm{ind}} + \vec{E}_{\mathrm{coul}} = 0$$

As I increases, $\vec{B}$ increases and the induced $\vec{E}$ opposes the increase, as shown. The induced field pushes charge until the Coulomb field is the exact opposite, leaving $\vec{E}_{\mathrm{total}} = 0$ inside. Then the potential decreases in the same direction as I, as also shown in the diagram..



$$\Delta V = L\frac{dI}{dt}$$

All these expressions remain correct whether the circuit variables Q, I and dI/dt are positive or negative.