INTRODUCTION

So far, the only information we have been able to get about the universe beyond the solar system is from the electromagnetic radiation that reaches us (and a few cosmic rays). So doing Astrophysics is like a big puzzle: look at the radiation and use it to figure out– well, everything! Where is the object, what is it, when and how did it form, how has it evolved since then, how does it fit in the "astrophysical zoo"?. So our objective in this course is to learn some of the tricks for solving the puzzle.

We start by looking at the way that radiation is modified as it travels from its source to the observer (us).

Radiative transfer

Vocabulary

Intensity

You can think of intensity as measuring the energy traveling along a light ray. Of course a single ray carries no energy, so we look at a bundle of rays within a solid angle d of the central ray. The energy crossing area dA normal to the ray in time dt and frequency range $d\nu$ is

$$dE = I_\nu dA dt d\nu d \tag{1}$$

The intensity I_ν =energy per unit time per unit frequency per unit area per unit solid angle units (astronomers like cgs units)

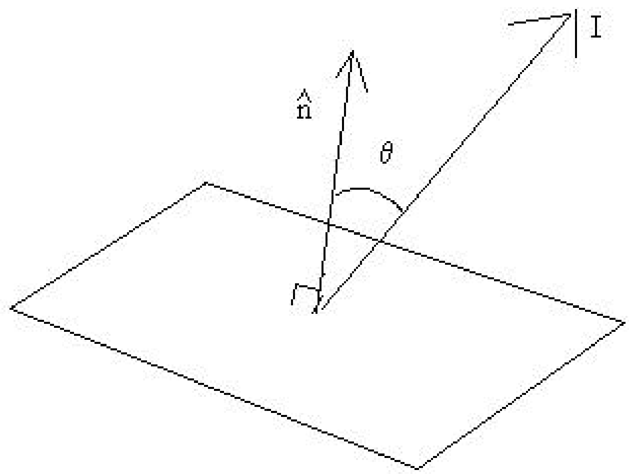
$$\frac{\mathrm{erg}}{\mathrm{sec} \cdot \mathrm{cm}^2 \cdot \mathrm{Hz} \cdot \mathrm{ster}}$$

Mean intensity J_n (ergs/cm^2·s·Hz) is obtained by averaging over angles:

$$J_\nu = \frac{1}{4\pi} \int I_\nu \, d \tag{2}$$

An isotropic radiation field has $J_\nu = I_\nu$.

Flux F_ν measures the energy crossing a surface, in erg/cm^2·s·Hz. To get the energy crossing the surface, we include only the component of the direction vector along the ray that is normal to the surface. That introduces a factor of $\cos\theta$.



$$F_\nu = \int I_\nu \cos\theta \; d \tag{3}$$

Flux is usually the easiest quantity to measure for a typical astrophysical source, such as a star. We can only measure intensity if the source is resolved, like the sun or the moon. The flux at Earth due to an unresolved source with luminosity L erg/s a distance D away is

$$F = \frac{L}{4\pi D^2}$$

Momentum flux and *radiation pressure*. The momentum of a photon is its energy/c, so we can get momentum flux from energy flux. Pressure is normal force/unit area=rate of change of momentum normal to the surface/area. So we compute the total momentum crosing area dA in time dt due to a bundle of rays at θ, ϕ as

$$d\vec{p}(\theta, \phi) = \frac{dF_\nu}{c}\hat{\imath}\; dAdt = \frac{I_\nu \cos\theta}{c} d \quad \hat{\imath}\; dAdt$$

where $\hat{\imath}$ is a unit vector along the ray. The normal component is

$$dp_{\rm normal}(\theta, \phi) = dp\cos\theta = \frac{dF_\nu}{c} dAdt\cos\theta = \frac{I_\nu \cos^2\theta}{c} d \quad dAdt$$

and integrating over the whole radiation field, we have

$$dp_{\rm normal} = \int \frac{I_\nu \cos^2\theta}{c} d \quad dAdt$$

and so the pressure on the surface patch is

$$P_\nu = \frac{dp_{\text{normal}}}{dAdt} = \frac{1}{c}\int I_\nu \cos^2\theta \quad d \tag{4}$$

Finally the total radiation energy in a volume $dV = dAd\ell$ contributed by radiation moving in direction θ, ϕ is

$$dE = du_\nu\left(\theta,\phi\right) dAd\ell d\nu$$

All the energy passes out of the volume in a time $dt = d\ell/c$, so from (1),

$$dE = I_\nu dA\frac{d\ell}{c}d\nu d$$

and comparing the two expressions we find

$$du_\nu\left(\theta,\phi\right) = \frac{I_\nu}{c}d$$

To get the total energy density we sum the contributions from rays in all directions, to get

$$u_\nu = \int du_\nu\left(\theta,\phi\right) = \int \frac{I_\nu}{c}d \quad = \frac{4\pi}{c}J_\nu \tag{5}$$

Let's use these definitions in a few simple examples.

Radiation pressure in a perfectly reflecting enclosure with an isotropic radiation field.

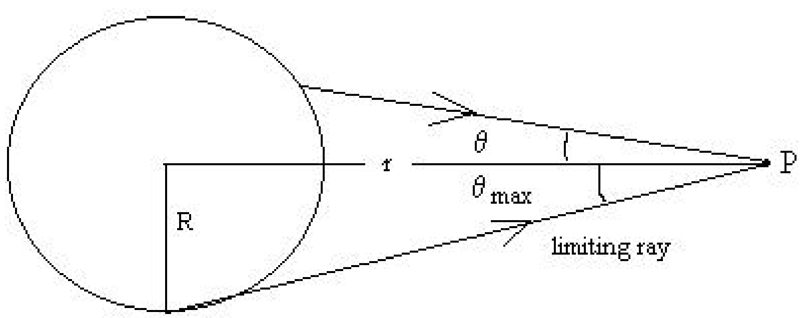
At each reflection the momentum tranferred is twice the incident momentum, so

$$P_\nu = \frac{2}{c}\int I_\nu \cos^2\theta d$$

But an isotropic radiation field means that I_ν is independent of direction, so $I_\nu = J_\nu$ and it comes out of the integral. Radiation only travels over half the total solid angle since no radiation is coming in from outside the enclosure:

$$P_\nu = \frac{2}{c}J_\nu\int_0^{+1}\mu^2 d\mu\int_0^{2\pi}d\phi = \frac{4\pi J_\nu}{c}\left.\frac{\mu^3}{3}\right|_0^{+1} = \frac{4\pi}{3c}J_\nu = \frac{1}{3}u_\nu$$

Flux from a uniformly bright sphere:



Rays reaching P leave the sphere at angles from $\theta = 0$ to $\theta = \theta_{\max}$, as shown. The intensity is the same along all rays. The flux at P is

$$F = \int I \cos\theta d \quad = 2\pi I \int_{\cos\theta_{\max}}^{1} \mu d\mu = \pi I \left(1 - \cos^2\theta_{\max}\right) = \pi I \sin^2\theta_{\max} = \pi I \left(\frac{R}{r}\right)^2$$

where in the last step we used the fact that $\sin\theta \simeq \tan\theta$ for small angles.

Notice that we cannot use the result for $r = R$ (as is done in the book) because that violates the small angle approximation we made. But at the surface the integral becomes

$$F = 2\pi I \int_{0}^{+1} \mu d\mu = \pi I$$

So the textbook result is correct even though their method of getting it is not.

Radiative Transfer.

So let's start with the intensity and look at the ray crossing a cylinder of area dA and thickness ds. As the ray crosses the cylinder, the intensity will increase due to emission in the cylinder, and decrease due to absorption. (We will omit scattering for the moment, and come back to that later.) The emission coefficient is j_ν (erg/cm^3·s·Hz) and the absorption may be written as the density of particles in the slab n $\left(\mathrm{cm}^{-3}\right)$times the absorption cross section per particle σ_ν (cm/Hz), or just as $\alpha_\nu = n\sigma_\nu$ $\left(\mathrm{cm}^{-1}\mathrm{Hz}^{-1}\right)$.

There are $ndAds$ particles in the slab with a total area $A_{\mathrm{part}} = n\sigma dAds$. The photons hitting that area will be absorbed, so

$$\text{Energy out} = \text{energy in} + \text{emission - absorption}$$

Dividing a factor of $d \quad dtd\nu$ from each term, we have

$$dAI_\nu\left(s + ds\right) = dAI_\nu\left(s\right) + j_\nu dAds - \alpha_\nu I_v dAds$$

or

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu \tag{6}$$

This is the equation of radiative transfer. To simplify, we define the *optical depth* τ_ν by

$$d\tau_\nu = \alpha_\nu ds$$

so the radiative transfer equation may be written as

$$\frac{dI_\nu}{d\tau} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

where

$$S_\nu \equiv \frac{j_\nu}{\alpha_\nu} \tag{7}$$

is the source function.

We may solve the equation formally as follows. Multiply the equation by e^{τ_v} and rearrange.

$$\frac{d}{d\tau_\nu}\left(e^{\tau_v} I_\nu\right) = e^{\tau_v} I_\nu + e^{\tau_v}\frac{dI_\nu}{d\tau_\nu} = e^{\tau_\nu} S_\nu$$

Thus

$$I_\nu\left(\tau_\nu\right) = e^{-\tau_\nu} I_\nu\left(0\right) + e^{-\tau_\nu}\int_0^{\tau_\nu} e^{\tau'_\nu} S_\nu\left(\tau'_\nu\right) d\tau'_\nu \qquad (8)$$