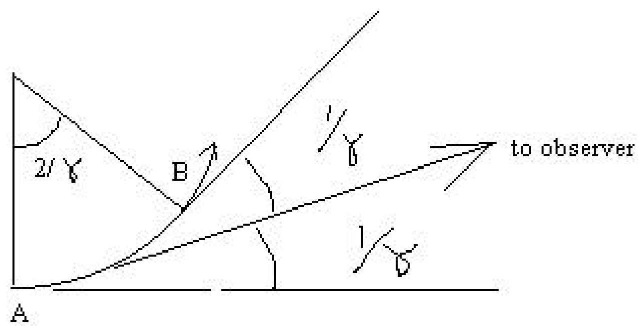
1 Synchrotron radiation

1.1 Qualitative discussion

Consider a particle moving along a circular path. The radiation is beamed into a cone of opening angle $\sim 1/\gamma$, as shown above. Thus an observer at P sees radiation while the particle is pointed within an angle $\sim 1/\gamma$ of P, and thus while the particle travels an arc of length $s \sim \frac{2}{\gamma} r$, where r is the radius of the circle. The particle travels this distance in a time

$$\Delta t = \frac{2}{\gamma}\frac{r}{v}$$



Radiation emitted at A at time t_1 reaches P, distance d away, at time $T_1 = \frac{d}{c} + t_1$.

Radiation emitted at B at time $t_2 = t_1 + \Delta t = t_1 + 2r/\gamma v$ reaches P at time $T_2 = \frac{d}{c} - \frac{2r}{\gamma c} + t_1 + \frac{2r}{\gamma v}$. The length of the observed pulse is

$$\Delta T = T_2 - T_1 = \frac{2r}{\gamma}\left(\frac{1}{v} - \frac{1}{c}\right)$$

Now $\gamma^2 = \frac{1}{1-\beta^2}$ and so $\beta^2 = 1 - \frac{1}{\gamma^2}$ and then:

$$\frac{1}{\beta} = \left(1 - \frac{1}{\gamma^2}\right)^{-1/2} \simeq 1 + \frac{1}{2\gamma^2}$$

Thus:

$$\Delta T = \frac{2r}{\gamma c}\left(\frac{1}{\beta} - 1\right) = \frac{2r}{\gamma c}\left(\frac{1}{\beta} - 1\right) = \frac{r}{\gamma^3 c}$$

But for an extremely relativistic particle, $\omega_0 = v/r \simeq c/r$, so $\Delta T = 1/\gamma^3\omega_0$. Now a pulse of width ΔT has Fourier components up to at least $1/\Delta T$, and so the observed frequencies extend to at least $\gamma^3\omega_0$, where $\omega_0 = eB/\gamma mc = \omega_B/\gamma$.

An exact calculation (See eg Jackson Ch 14) shows that the spectrum actually peaks around this high frequency $\nu \approx \gamma^2 \nu_B$. We can find a typical value for

ν_B :

$$\nu_B = \frac{eB}{2\pi mc} = \frac{4.8\times10^{-10}\text{esu}\times1\times10^{-6}\text{ G}}{2\pi\,(511\text{keV}/c)}B_{\mu G}$$

Units: In this unit system

$$1\text{G} = \frac{1\text{ esu}}{(1\text{ cm})^2}$$

and

$$1\text{ erg} = \frac{1\text{ esu}^2}{1\text{ cm}}$$

So

$$\begin{aligned}\nu_B &= \frac{4.8\times10^{-10}\text{esu}\times1\times10^{-6}\text{ G}}{2\pi\,(511\times1.6\times10^{-9}\text{erg})}3\times10^{10}\text{ cm/s }B_{\mu G}\\ &= \frac{4.8\times10^{-10}\text{esu}\times1\times10^{-6}\text{ esu/cm}^2}{2\pi\,(511\times1.6\times10^{-9}\text{erg})}3\times10^{10}\text{ cm/s }B_{\mu G}\\ &= 2.8\text{ Hz }B_{\mu G} \qquad (1)\end{aligned}$$

This is a useful number to remember.

For cyclotron radiation, the power radiated is

$$\begin{aligned}P &= \frac{2}{3}\frac{e^2}{c^3}a^2 = \frac{2}{3}\frac{e^2}{c^3}\left(\frac{evB_\perp}{mc}\right)^2\\ &= \frac{2}{3}\frac{e^4v^2}{m^2c^5}B_\perp^2\end{aligned}$$

For relativistic particles, $v\simeq c$, we can obtain the correct result by writing the formula in covariant form. Since $P\sim E/t$ and both E and t are $0-$components of 4-vectors, we may write

$$P = -\frac{2}{3}\frac{e^2}{c^3}\overleftrightarrow{a}\cdot\overleftrightarrow{a}$$

where

$$\overleftrightarrow{a} = \frac{d\overleftrightarrow{v}}{d\tau} = \gamma\left(c\frac{d\gamma}{dt},\frac{d}{dt}(\gamma v)\right)$$

and

$$\frac{d\gamma}{dt} = \frac{d}{dt}\frac{1}{\sqrt{1-\vec{v}\cdot\vec{v}/c^2}} = \gamma^3\vec{\beta}\cdot\frac{\vec{a}}{c}$$

Thus

$$\begin{aligned}-\overleftrightarrow{a}\cdot\overleftrightarrow{a} &= -\gamma^2\left[\left(c\frac{d\gamma}{dt}\right)^2 - v^2\left(\frac{d\gamma}{dt}\right)^2 - \gamma^2a^2\right]\\ &= \gamma^2\left[\gamma^2a^2 - c^2\left(\gamma^3\vec{\beta}\cdot\frac{\vec{a}}{c}\right)^2(1-\beta^2)\right]\\ &= \gamma^4\left[a^2-\gamma^2\left(\vec{\beta}\cdot\vec{a}\right)^2\right]\end{aligned}$$

For synchrotron radiation with $\vec{a} \perp \vec{v}$,

$$-\overleftrightarrow{a} \cdot \overleftrightarrow{a} = \gamma^4 a^2$$

and thus

$$\begin{aligned} P &= \frac{2}{3}\frac{e^2}{c^3}\gamma^4 \left(\frac{evB_\perp}{\gamma mc}\right)^2 \\ &= \frac{2}{3}\frac{e^4}{m^2c^3}\gamma^2 B_\perp^2 \end{aligned}$$

If $\vec{v}$ makes an angle α with the magnetic field, then $B_\perp = B\sin\alpha$ and

$$P = \frac{2}{3}\frac{e^4}{m^2c^3}\gamma^2 B^2 \sin^2\alpha$$

For an isotropic distribution of particle velocities

$$< \sin^2\alpha >= \frac{1}{2}\int_{-1}^{+1} \left(1-\mu^2\right) d\mu = \frac{2}{3}$$

Thus

$$P \sim \frac{4}{9}\frac{8\pi e^4}{m^2c^3}\gamma^2 \frac{B^2}{8\pi} = \frac{4}{3}\sigma_T c \gamma^2 u_B$$

This should be compared with the result for Compton scattering (eqn 1 in those notes)

$$P = \frac{4}{3}\sigma_T c\gamma^2 u_{\rm ph}$$

Synchrotron radiation may be viewed as scattering of virtual photons associated with the magnetic field.

1.2 Spectrum of emitted radiation

1.3 For a power law distribution of electrons, we may compute the radiated spectrum as we did for compton scattering. We approximate by assuming that all the emission is at the peak frequency, and using a delta function.

$$
\begin{aligned}
j_\nu &= \frac{1}{4\pi}\int P_\nu(\gamma)\, n(\gamma)\, d\gamma = \frac{1}{4\pi}\int \frac{4}{3}\sigma_T c\gamma^2 u_B \delta\left(\nu - \gamma^2\nu_B\right) K\gamma^{-p} d\gamma \\
&= K\frac{\sigma_T c}{3\pi}u_B \int \gamma^2 \frac{\delta\left(\gamma - \sqrt{\nu/\nu_B}\right)}{2\gamma\nu_B}\gamma^{-p} d\gamma \\
&= K\frac{\sigma_T c}{3\pi}u_B \frac{1}{2\nu_B}\left(\sqrt{\frac{\nu}{\nu_B}}\right)^{-p+1} \\
&= K\frac{\sigma_T c}{3\pi}u_B \frac{\nu_B^{(p-3)/2}}{2}\nu^{-(p-1)/2} \\
&= K\frac{8\pi e^4}{3m^2c^3}\frac{c}{6\pi}\frac{B^2}{8\pi}\left(\frac{eB}{2\pi mc}\right)^{(p-3)/2}\nu^{-(p-1)/2} \\
&= K\frac{e^4}{18\pi m^2c^2}\left(\frac{e}{2\pi mc}\right)^{(p-3)/2} B^{(p+1)/2}\nu^{-(p-1)/2}
\end{aligned}
$$

Thus the emiited spectrum is also a power law whose slope reflects the slope of the underlying electron spectrum.

$$j_\nu \propto \nu^{-\alpha} \quad \textbf{with} \quad \alpha = \frac{p-1}{2}$$

The emission is also proportional to the electron density (through K) and to the magnetic field strength

$$j_\nu \propto B^{(p+1)/2} = B^{\alpha+1}$$

Astrophysical sources of synchrotron radiation

1.3.1 Supernova remnants

Some supernova remnants have a shell structure- we'll talk about these more when we get to fluids and shock waves. But some look "filled" on the sky. These sources are called plerions. The prototypical source is the Crab Nebula. These sources typically exhibit power-law spectra in the radio with spectral index $\alpha \sim 0.8$. The pulsar (remnant of the exploded star) accelerates electrons to high energy, and these particles produce synchroton radiation. The crab radiates all the way out to gamma rays.

If all the particles are accelerated in the initrial explosion, then as the remnant expands the density of particles (and also the magnetic field) decreases. This leads to a dimming of the source. The resulting relation between synchrotron surface brightness and size was used to determine distances. However there are pitfalls because there are different phases in SNR evolution, as we'll see.

An electron of energy γmc^2 loses energy at a rate P and so has a lifetime

$$\begin{aligned}\tau &= \frac{\gamma mc^2}{P} = \frac{\gamma mc^2}{\frac{4}{9}\frac{e^4}{m^2c^3}\gamma^2 B^2} = \frac{9}{4\gamma}\frac{m^3c^5}{e^4B^2} = \frac{9}{4\gamma}\frac{\left(mc^2\right)^3}{ce^4B^2} \\ &= \frac{1}{\gamma}\frac{9}{4}\frac{(511\text{ keV})^3}{(3\times 10^{10}\text{ cm/s})\,(4.8\times 10^{-10}\text{ esu})^4\,(10^{-6}\text{ G})^2 B_{\mu G}^2}\end{aligned}$$

So

$$\begin{aligned}\tau &= \frac{1}{\gamma}\frac{9}{4}\frac{(511\text{ keV})^3\left(1.6\times 10^{-9}\text{ erg/keV}\right)^3}{(3\times 10^{10}\text{ cm/s})\,(4.8\times 10^{-10}\text{ esu})^4\left(10^{-6}\text{ esu/cm}^2\right)^2 B_{\mu G}^2} \\ &= \frac{1}{\gamma B_{\mu G}^2}7.721\,8\times 10^{20}\frac{\text{erg}^3}{\text{esu}^6}\text{cm}^3\cdot\text{s} \\ &= \frac{1}{\gamma B_{\mu G}^2}\frac{7.7\times 10^{20}\text{s}}{\pi\times 10^7\text{ s/y}} = \frac{1}{\gamma B_{\mu G}^2}2.5\times 10^{13}\text{ y}\end{aligned} \tag{2}$$

The value of γ needed to produce a photon of energy $\mathcal{E}$ is given by

$$\gamma^2\frac{eB}{2\pi mc} = \nu = \frac{\mathcal{E}}{h}$$

So

$$\begin{aligned}\gamma &= \sqrt{\frac{\mathcal{E}}{h}\frac{2\pi mc}{eB}} = \sqrt{\frac{\mathcal{E}}{hc}\frac{2\pi mc^2}{eB}} \\ &= \sqrt{\frac{\mathcal{E}_{\text{keV}}}{B_{\mu G}}}\sqrt{\frac{2\pi\,(511\text{ keV})\,(1\text{ keV})}{(1.24\text{ keV}\cdot\text{nm})\,(4.8\times 10^{-10}\text{ esu})\left(10^{-6}\text{ esu/cm}^2\right)}} \\ &= \sqrt{\frac{\mathcal{E}_{\text{keV}}}{B_{\mu G}}}\sqrt{\frac{2\pi\,(511)\,(1.6\times 10^{-9}\text{ erg})}{(1.24\ \times 10^{-7}\text{cm})\,(4.8\times 10^{-10}\text{ esu})\left(10^{-6}\text{ esu/cm}^2\right)}} \\ &= \sqrt{\frac{\mathcal{E}_{\text{keV}}}{B_{\mu G}}}2.9\times 10^8\end{aligned} \tag{3}$$

Thus the lifetime of the particle in terms of the energy of the emitted photon is

$$\begin{aligned}\tau &= \frac{1}{B_{\mu G}^2}\frac{2.5\times 10^{13}\text{ y}}{2.9\times 10^8}\sqrt{\frac{B_{\mu G}}{\mathcal{E}_{\text{keV}}}} \\ &= \frac{8.6\times 10^4\text{ y}}{\sqrt{B_{\mu G}^3\mathcal{E}_{\text{keV}}}}\end{aligned}$$

So if we observe $\gamma-$rays with energy 10 MeV, the lifetime is

$$\tau \sim \frac{8.6 \times 10^4 \text{ y}}{\sqrt{10^4}\sqrt{B_{\mu G}^3}} = 860 \text{ y}\frac{1}{\sqrt{B_{\mu G}^3}}$$

Since the crab was formed in 1054, 950 years ago, we conclude that energetic particles must be continually supplied to the remnant.

1.3.2 Extragalactic radio sources

These are most often associated with giant elliptical galaxies, and they are double-lobed. Each lobe is fed by a jet that runs from the central engine at the nucleus out to the lobe. The specturm is a power law with $\alpha \sim 1$. Letting $E = \gamma mc^2$ be the electron energy, the total luminosity is

$$\begin{aligned} L &= \int P(\gamma) N(\gamma) d\gamma \\ &= \int_{\gamma_{\min}}^{\gamma_{\max}} \frac{4}{3}\sigma_T c\gamma^2 u_B K\gamma^{-p} d\gamma \\ &= \frac{4}{3}\sigma_T c u_B K \left.\frac{\gamma^{3-p}}{3-p}\right]_{\gamma_{\min}}^{\gamma_{\max}} = \frac{4}{3}\sigma_T c u_B K \frac{\gamma_{\max}^{3-p} - \gamma_{\min}^{3-p}}{3-p} \end{aligned} \tag{4}$$

We can obtain $\gamma_{\max}$ from the maximum frequency in the observed spectrum. Then (equation1)

$$\gamma_{\max} = \sqrt{\frac{2\pi\nu_{\max} mc}{eB}} = \sqrt{\frac{\nu_{\max}}{3\,(\text{Hz}/\mu\text{G})\,B_{\mu G}}}$$

Then since $\alpha = (p-1)/2$, $p-1 = 2\alpha$ and $3-p = 2(1-\alpha)$:

$$\begin{aligned} L &= \frac{4}{3}\sigma_T c\frac{B^2}{8\pi}K\left(\frac{2\pi mc}{eB}\right)^{(3-p)/2} \frac{\nu_{\max}^{(3-p)/2} - \nu_{\min}^{(3-p)/2}}{3-p} \\ &= \frac{1}{3}\sigma_T c\left(\frac{2\pi mc}{e}\right)^{1-\alpha} \frac{K}{2\pi}B^{1+\alpha}\left[\frac{\nu_{\max}^{1-\alpha} - \nu_{\min}^{1-\alpha}}{2(1-\alpha)}\right] \\ &= \frac{1}{3}\sigma_T c\left(\frac{mc}{e}\right)^{1-\alpha} \frac{K}{(2\pi)^{\alpha}}B^{1+\alpha}\left[\frac{\nu_{\max}^{1-\alpha} - \nu_{\min}^{1-\alpha}}{2(1-\alpha)}\right] \end{aligned} \tag{5}$$

The total energy in electrons is:

$$\begin{aligned} E_e &= \int \gamma mc^2 N(\gamma) d\gamma \\ &= Kmc^2 \left.\frac{\gamma^{2-p}}{2-p}\right|_{\gamma_{\min}}^{\gamma_{\max}} \\ &= Kmc^2\frac{\gamma_{\max}^{2-p} - \gamma_{\min}^{2-p}}{2-p} \end{aligned}$$

We can replace the unobservable parameter K with the observable Luminosity L, (equation 5) and $2-p = 1-2\alpha$

$$\begin{aligned} E_e &= mc^2 \frac{\gamma_{\max}^{2-p} - \gamma_{\min}^{2-p}}{2-p} \frac{L}{\frac{4}{3}\sigma_T c u_B \frac{\gamma_{\max}^{3-p} - \gamma_{\min}^{3-p}}{3-p}} \\ &= \frac{3}{4} 8\pi \frac{mc}{\sigma_T} \frac{\gamma_{\max}^{2-p} - \gamma_{\min}^{2-p}}{\gamma_{\max}^{3-p} - \gamma_{\min}^{3-p}} \frac{L}{B^2} \frac{3-p}{2-p} \\ &= 6\pi \frac{mc}{\sigma_T \gamma_{\max}} \frac{1-(\gamma_{\min}/\gamma_{\max})^{2-p}}{1-(\gamma_{\min}/\gamma_{\max})^{3-p}} \frac{L}{B^2} \frac{2(1-\alpha)}{1-2\alpha} \end{aligned}$$

The energy stored in the magnetic field in the source volume V is

$$U_B = \frac{B^2}{8\pi} V$$

Thus the total energy is

$$E_e + U_B = 6\pi \frac{mc}{\sigma_T \gamma_{\max}} \frac{1-(\gamma_{\min}/\gamma_{\max})^{1-2\alpha}}{1-(\gamma_{\min}/\gamma_{\max})^{2(1-\alpha)}} \frac{L}{B^2} \frac{2(1-\alpha)}{1-2\alpha} + \frac{B^2}{8\pi} V$$

Writing the result in terms of the observed frequencies, we have

$$E_e + U_B = 6\pi \sqrt{\frac{eB}{2\pi \nu_{\max} mc}} \frac{mc}{\sigma_T} \frac{1-(\nu_{\min}/\nu_{\max})^{(1-2\alpha)/2}}{1-(\nu_{\min}/\nu_{\max})^{(1-\alpha)}} \frac{L}{B^2} \frac{2(1-\alpha)}{1-2\alpha} + \frac{B^2}{8\pi} V$$

The first term is (roughly) inversely proportional to $B^{3/2}$ while the second is directly proportional to B^2.

$$E_{\text{tot}} = \frac{A}{B^{3/2}} + B^2 \frac{V}{8\pi}$$

with

$$A = \frac{3}{\sigma_T} \sqrt{\frac{2\pi emc}{\nu_{\max}}} \frac{1-(\nu_{\min}/\nu_{\max})^{(1-2\alpha)/2}}{1-(\nu_{\min}/\nu_{\max})^{(1-\alpha)}} \frac{2(1-\alpha)}{1-2\alpha} L \tag{6}$$

Thus we can minimize the energy required to produce the observed luminosity by adjusting B :

$$\frac{dE_{\text{tot}}}{dB} = -\frac{3}{2} \frac{A}{B^{5/2}} + \frac{BV}{4\pi} = 0 \Rightarrow B = \left(\frac{6\pi A}{V}\right)^{2/7} \tag{7}$$

With this value we find

$$U_B = \left(\frac{6\pi A}{V}\right)^{4/7} \frac{V}{8\pi} = \frac{6^{\frac{4}{7}}}{8} \frac{V^{3/7}}{\pi^{3/7}} A^{4/7}$$

while

$$E_e = A\left(\frac{6\pi A}{V}\right)^{-3/7} = 6^{\frac{3}{7}}\frac{V^{3/7}}{\pi^{3/7}}A^{4/7} = \frac{4}{3}U_B$$

Thus the least possible total energy is required if the energy is almost equally divided between the electrons and the magnetic field. This condition is referred to as "equipartition". Then the total energy is

$$\begin{aligned} E_{\rm tot} &= \frac{7}{4}\frac{V^{3/7}}{(6\pi)^{3/7}}A^{4/7} \\ &= \frac{7}{4}\frac{V^{3/7}L^{4/7}}{(6\pi)^{3/7}}\left[\frac{3}{\sigma_T}\sqrt{\frac{2\pi emc}{\nu_{\rm max}}}\frac{1-(\nu_{\rm min}/\nu_{\rm max})^{(1-2\alpha)/2}}{1-(\nu_{\rm min}/\nu_{\rm max})^{(1-\alpha)}}\frac{2(1-\alpha)}{1-2\alpha}\right]^{4/7} \end{aligned}$$

Example: Cygnus A

$\alpha = 0.75$, Flux $= 1.3\times10^4$ Jy @ 100 MHz. Angular size of each lobe is 38", distance is 220 Mpc. $\nu_{\rm min} \sim 1$ MHz and $\nu_{\rm max} \sim 1500$ MHz. Then

$$V = 2\left(\frac{4}{3}\pi r^3\right)$$

with

$$\begin{aligned} r &= \frac{1}{2}(220\ \text{Mpc})\times\frac{38}{60\times60}\times\frac{\pi}{180} \\ &= 2.0\times10^{-2}\ \text{Mpc} = 20\ \text{kpc} \end{aligned}$$

Thus

$$V = \frac{8}{3}\pi\left(20\times3\times10^{21}\ \text{cm}\right)^3 = 1.8\times10^{69}\ \text{cm}^3$$

and

$$\begin{aligned} L &= \int_{\nu_{\rm min}}^{\nu_{\rm max}} F_0\left(\frac{\nu}{\nu_0}\right)^{-\alpha} d\nu\times4\pi d^2 \\ &= \frac{F_0\nu_0}{1-\alpha}\left\{\left(\frac{\nu_{\rm max}}{\nu_0}\right)^{1-\alpha}-\left(\frac{\nu_{\rm min}}{\nu_0}\right)^{1-\alpha}\right\}\times4\pi d^2 \\ &= \frac{\left(1.3\times10^4\text{Jy}\right)\left(100\times10^6\ \text{Hz}\right)}{0.25}\left\{15^{.25}-\left(\frac{1}{10}\right)^{0.25}\right\}\times4\pi\,(220\ \text{Mpc})^2 \\ &= \frac{\left(1.3\times10^4\times10^{-26}\ \text{W/m}^2\cdot\text{Hz}\right)\left(100\times10^6\ \text{Hz}\right)}{0.25}\{1.4\}\times4\pi\left(220\times3\times10^{22}\ \text{m}\right)^2 \\ &= 3.985\times10^{37}\ \text{W} = 4\times10^{44}\ \text{erg/s} \simeq 10^{11}\ \text{L}_\odot \end{aligned}$$

Then

$$
\begin{aligned}
B_{\mathrm{eq}} &= \left(\frac{6\pi A}{V}\right)^{2/7} \\
&= \left(\frac{6\pi}{1.8\times10^{69}\ \mathrm{cm}^3}\right)^{2/7}\left[\frac{3}{\sigma_T}\sqrt{\frac{2\pi emc}{\nu_{\max}}}\frac{1-(\nu_{\min}/\nu_{\max})^{(1-2\alpha)/2}}{1-(\nu_{\min}/\nu_{\max})^{(1-\alpha)}}\frac{2(1-\alpha)}{1-2\alpha}L\right]^{2/7} \\
&= \left(\frac{6\pi}{1.8\times10^{69}\ \mathrm{cm}^3}\right)^{2/7}\left[\frac{3}{0.66\times10^{-24}\ \mathrm{cm}^2}\frac{1-(1/1500)^{-1/4}}{1-(1/1500)^{1/4}}\frac{0.5}{-0.5}4\times10^{44}\ \mathrm{erg/s}\right]^{2/7}\left(\frac{2\pi emc}{\nu_{\max}}\right)^{1/7} \\
&= 3.9126\left(\frac{2\pi\times4.8\times10^{-10}\ \mathrm{esu}\ 9\times10^{-28}\ \mathrm{g}\ 3\times10^{10}\mathrm{cm/s}}{1500\times10^{6}\ \mathrm{Hz}}\right)^{1/7}\left(\frac{\mathrm{erg/s}}{\mathrm{cm}^5}\right)^{2/7} \\
&= 5\times10^{-5}\left(\frac{\mathrm{esu}^2}{\mathrm{cm}^6\cdot\mathrm{s}}\right)^{2/7}(\mathrm{esu}\cdot\mathrm{g}\cdot\mathrm{cm})^{1/7}
\end{aligned}
$$

Check the units:

$$
\begin{aligned}
\left(\frac{\mathrm{esu}^2}{\mathrm{cm}^6\cdot\mathrm{s}}\right)^{2/7}(\mathrm{esu}\cdot\mathrm{g}\cdot\mathrm{cm})^{1/7} &= \left(\frac{\mathrm{esu}^5\cdot\mathrm{g}\cdot\mathrm{cm}}{\mathrm{cm}^{12}\cdot\mathrm{s}^2}\right)^{1/7} = \left(\frac{\mathrm{esu}^5\cdot\mathrm{erg/cm}}{\mathrm{cm}^{12}}\right)^{1/7} \\
&= \left(\frac{\mathrm{esu}^5\cdot\mathrm{esu}^2/\mathrm{cm}}{\mathrm{cm}^{14}}\right)^{1/7} = \frac{\mathrm{esu}}{\mathrm{cm}^2} = \mathrm{G}
\end{aligned}
$$

Whew! Thus the equipartition field is about 50 μG. The minimum energy is then

$$
\begin{aligned}
E_{\mathrm{tot}} &= \frac{7}{3}U_B = \frac{7}{3}\frac{B_{\mathrm{eq}}^2}{8\pi}V \\
&= \frac{7}{3}\frac{\left(5\times10^{-5}\ \mathrm{G}\right)^2}{8\pi}\left(1.8\times10^{69}\ \mathrm{cm}^3\right) \\
&= 4.\times10^{59}\ \mathrm{erg}
\end{aligned}
$$

and the γ we need to get a 1500 MHz photon is

$$
\gamma_{\max} = \sqrt{\frac{\nu_{\max}}{3\,(\mathrm{Hz}/\mu\mathrm{G})\,B_{\mu G}}} = \sqrt{\frac{1500\times10^6}{3\times50}} = 3.\times10^3
$$

and the lifetime of this electron is (equation2):

$$
\begin{aligned}
\tau &= \frac{1}{\gamma B_{\mu G}^2}2.5\times10^{13}\ \mathrm{y} = \frac{1}{3000\,(50)^2}2.5\times10^{13}\ \mathrm{y} \\
&= 3.\times10^{6}\ \mathrm{y}
\end{aligned}
$$

Thus sources like Cygnus A are either very short-lived (by astronomical standards) or the relativistic particles must be resupplied by reacceleration in situ or transport from a central engine.

The jets visible in this radio image may be the source of resupply, but, if so, it is an inefficient process. (We can see the jets, therefore they radiate away some of the transported energy.)

When observations of B can be made, it appears that B is less than the equipartition value. This increases the energy requirements but also increases the source lifetime.