**More on thermal instability**

Let the cooling be due to
(1) A line that radiates most at a tempertaure $T_0$.

$$\Lambda_{\rm line} = n\mathcal{L}_{\rm line} \sim An \exp\left[-\left(\frac{T}{T_0} - 1\right)^2\right]$$

(2) Bremmsstrahlung

$$\Lambda_{\rm Br} = n\mathcal{L}_{\rm Br} \sim Bn^2T^{1/2}$$

Thus

$$\mathcal{L} \sim A\exp\left[-\left(\frac{T}{T_0} - 1\right)^2\right] + BnT^{1/2}$$

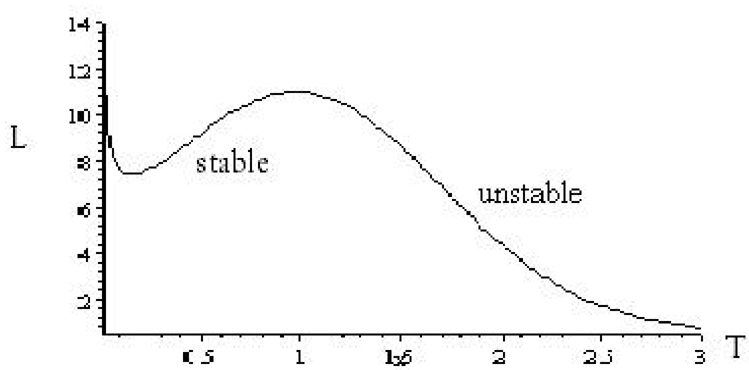
At constant pressure

$$n = \frac{P}{kT}$$

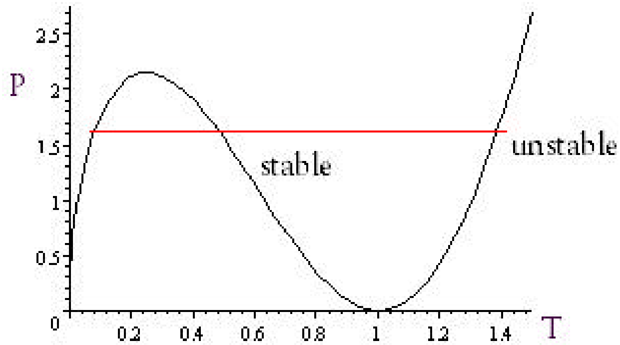
So

$$\mathcal{L} \sim \left\{A\exp\left[-\left(\frac{T}{T_0} - 1\right)^2\right] + \left(\frac{P}{kT}\right)BT^{1/2}\right\}$$

This function looks like
$10\exp\left(-(x-1)^2\right) + 1/\sqrt{x}$

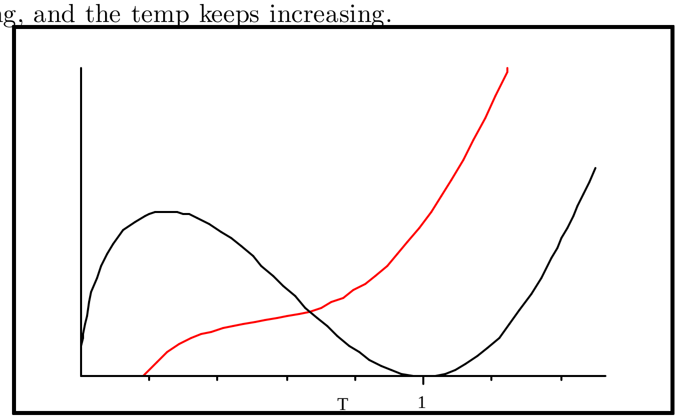



This graph shows $P$ versus $T$ at constant $\mathcal{L}$.




There are three solutions at the same pressure and $\mathcal{L}$, but only one is stable.

Here's another way to look at it. The red curve is greater $\mathcal{L}$. In the central region, moving to higher temperature increases the cooling, so the temperature drops again. On the far right, lower temperature gets to a region of more cooling, and the temperature keeps dropping. On the far left, higher temp implies less cooling, and the temp keeps increasing.




At constant entropy:

$$P \sim \rho^{\gamma} \quad \text{or } T \sim \rho^{\gamma-1} \sim \rho^{2/3}$$

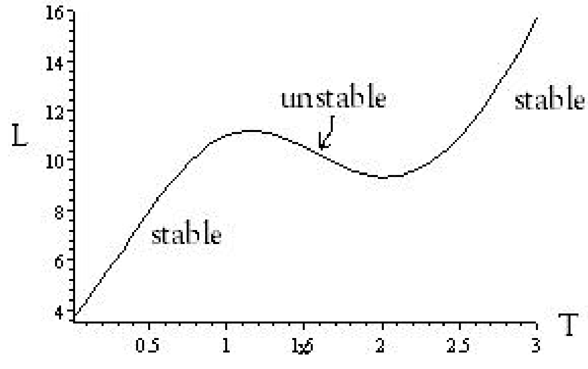
so

$$n \sim T^{3/2}$$

Thus

$$\mathcal{L} \sim A \exp\left[-\left(\frac{T}{T_0} - 1\right)^2\right] + BT^{5/2}$$

$10\exp\left(-(x-1)^2\right) + x^{5/2}$



16
14
12
10
8
6
4
L
unstable
stable
stable
0.5
1
1.5
2
2.5
3
T