Summing the original series for V :

$$V(x,y) = \frac{4V_0}{\pi} \sum_{\text{n odd}} \frac{1}{n} e^{-n\pi x/a} \sin \frac{n\pi y}{a}$$

Note that

$$\begin{aligned} \frac{1}{n} e^{-n\pi x/a} \sin \frac{n\pi y}{a} &= \operatorname{Im} \left(\frac{1}{n} e^{-n\pi x/a} \exp\left(i \frac{n\pi y}{a} \right) \right) \\ &= \operatorname{Im} \frac{z^n}{n} \end{aligned}$$

where

$$z = \exp\left(\frac{\pi}{a} (-x + iy) \right) = \exp\left(-\frac{\pi}{a} (x - iy) \right) = \exp\left(-\frac{\pi}{a} w \right)$$

where

$$w = x - iy$$

Thus the sum is

$$\begin{aligned} \operatorname{Im} \sum_{n \text{ odd}} \frac{z^n}{n} &= \operatorname{Im} \sum_{n \text{ odd},=1}^{\infty} \int z^{n-1} dz = \operatorname{Im} \int \sum_{n=0, \text{ even}}^{\infty} z^n dz \\ &= \frac{1}{2} \operatorname{Im} \int \left\{ \frac{1}{1-z} + \frac{1}{1+z} \right\} dz \\ &= \frac{1}{2} \operatorname{Im} \left\{ \ln(1+z) - \ln(1-z) \right\} \\ &= \frac{1}{2} \operatorname{Im} \ln \frac{1+z}{1-z} \\ &= \frac{1}{2} \operatorname{Im} \ln \frac{1 + \exp\left(-\frac{\pi}{a} w\right)}{1 - \exp\left(-\frac{\pi}{a} w\right)} = \frac{1}{2} \arg \frac{1 + \exp\left(-\frac{\pi}{a} w\right)}{1 - \exp\left(-\frac{\pi}{a} w\right)} \end{aligned}$$

Now

$$\begin{aligned} \frac{1 + \exp\left(-\frac{\pi}{a} w\right)}{1 - \exp\left(-\frac{\pi}{a} w\right)} &= \frac{1 + \exp\left(-\frac{\pi}{a} (x - iy)\right)}{1 - \exp\left(-\frac{\pi}{a} (x - iy)\right)} = \frac{1 + e^{-\pi x/a} e^{i\pi y/a}}{1 - e^{-\pi x/a} e^{i\pi y/a}} \\ &= \frac{1 + e^{-\pi x/a} e^{i\pi y/a}}{1 - e^{-\pi x/a} e^{i\pi y/a}} \frac{1 - e^{-\pi x/a} e^{-i\pi y/a}}{1 - e^{-\pi x/a} e^{-i\pi y/a}} \\ &= \frac{1 - e^{-\pi \frac{x}{a}} e^{-i\pi \frac{y}{a}} + e^{-\pi \frac{x}{a}} e^{i\pi \frac{y}{a}} - e^{-2\pi \frac{x}{a}}}{1 - e^{-\pi \frac{x}{a}} e^{-i\pi \frac{y}{a}} - e^{-\pi \frac{x}{a}} e^{i\pi \frac{y}{a}} + e^{-2\pi \frac{x}{a}}} \\ &= \frac{1 + e^{-\pi \frac{x}{a}} 2i \sin \frac{\pi y}{a} - e^{-2\pi \frac{x}{a}}}{1 - e^{-\pi \frac{x}{a}} 2 \cos \frac{\pi y}{a} + e^{-2\pi \frac{x}{a}}} \end{aligned}$$

and so

$$\begin{aligned} \arg \frac{1 + e^{-\pi \frac{x}{a}} 2i \sin \pi \frac{y}{a} - e^{-2\pi \frac{x}{a}}}{1 - e^{-\pi \frac{x}{a}} 2 \cos \pi \frac{y}{a} + e^{-2\pi \frac{x}{a}}} &= \tan^{-1} \frac{e^{-\pi \frac{x}{a}} 2 \sin \pi \frac{y}{a}}{1 - e^{-2\pi \frac{x}{a}}} \\ &= \tan^{-1} \frac{\sin \pi \frac{y}{a}}{\sinh \pi \frac{x}{a}} \end{aligned}$$

Then

$$V = \frac{4V_0}{\pi}\frac{1}{2}\tan^{-1}\frac{\sin\pi\frac{y}{a}}{\sinh\pi\frac{x}{a}} = \frac{2V_0}{\pi}\tan^{-1}\frac{\sin\pi\frac{y}{a}}{\sinh\pi\frac{x}{a}}$$

as required.